DSB – 230

Second Year B.Sc. Degree Examination, Sept./Oct. 2012
Directorate of Distance Education
MATHEMATICS (Paper – II)

Time : 3 Hours Max. Marks : 90

***Note**: Answer **any SIX** full questions of the following choosing atleast **ONE** from **each** Part.*

PART – A

1. a) i) Find order and degree of differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$. **2**

   ii) Solve $y = x.\left(dy/dx\right) + \frac{1}{dy/dx}$. **2**

   b) Solve $(x^2 + 2y^2)\,dx - xy.dy = 0$ given that $y = 0$ when $x = 1$. **5**

   c) Solve $x.\frac{dy}{dx} + (1-x)\,y = x^2y^2$. **6**

2. a) i) Solve $P^2 + (y - 2x)\,P - 2xy = 0$. **2**

   ii) Find General and Singular solution of the equation $P = \log(Px - y)$. **2**

   b) Solve $2Px - yP^2 = y$. **5**

   c) Find orthogonal trajectories of the family of curves $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$. Where '$\lambda$' is a parameter. **6**

PART – B

3. a) i) Solve $(D^3 - 2D^2 + 4D - 8)\,y = 0$, where $D = \frac{d}{dx}$. **2**

   ii) Solve $(D^2 + 4)\,y = \sin 2x$, where $D = \frac{d}{dx}$. **2**

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b) Solve $(D^2 - 2D + 1)\, y = x^2 e^{3x}$, where $D = \frac{d}{dx}$. **5**

c) Solve $(D^3 + 1)\, y = 5e^x . x^2$, where $D = \frac{d}{dx}$. **6**

4. a) i) Evaluate $\lim_{x \to 0} x . \log x$. **2**

ii) Evaluate $\lim_{x \to \infty} \frac{x . \cos 1/x}{1 + x}$. **2**

b) Expand $e^{\sin x}$ using Maclaurin's series up to the terms containing x^4. **5**

c) State and prove Lagrange's mean value theorem. **6**

PART – C

5. a) i) Show that in a group G, $(ab)^{-1} = b^{-1} . a^{-1}$. **2**

ii) Find the generators of a multiplicative group {1, –1, i, –i}. **2**

b) If 'a' is a generator of a cyclic group 'G', then prove that O(a) = O(G). **5**

c) If 'G' is any finite group, and H is any subgroup of G, then prove that O(H) divides O(G). **6**

6. a) i) Solve $2x - 3 < 5x + 3 < 2x + 3$. **2**

ii) Show that for any two real numbers x and y **2**

$| x + y | \leq | x | + | y |$

b) Find the order of the permutation and also find whether it is odd or even. **5**

$$\theta = \begin{pmatrix} a & b & c & d & e & f & g & h & i \\ b & c & d & a & f & g & e & i & h \end{pmatrix}$$

c) Find the envelop of family of circles $x^2 + y^2 - 2ax\cos\alpha - 2ay\sin\alpha = c^2$. Where '$\alpha$' is a parameter. **6**

PART – D

7. a) i) Find the limit of the sequence $\frac{6n-7}{5n+4}$. **2**

ii) Prove that $\lim_{n\to\infty} \frac{2n^2+8\sin(\pi/n)}{n^2} = 2$. **2**

b) If the sequence $\{X_n\}$ converges to 'l' and $\{y_n\}$ converges to 'm', then show that $\{x_n \times y_n\}$ converges to l×m. **5**

c) Show that the limit of a convergent sequence is unique. **6**

8. a) i) Show that $\sum \frac{n^3}{3^n}$ series is convergent. **2**

ii) Discuss the convergent of the series $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots$ **2**

b) State and prove Raabe's test. **5**

c) Find the sum to infinity of the following series : **6**

$$1+\frac{2}{6}+\frac{2.5}{6.12}+\frac{2.5.8}{6.12.18}+\ldots$$