Second Year B.Sc. Degree Examinations
December 2017
(Directorate of Distance Education)
MATHEMATICS
Paper – II: DSB 230: Mathematics

Time: 3 hrs]   *[Max. Marks: 90*

Instructions to candidates:

Answer any **SIX** full questions of the following choosing at least one from each part.

PART – A

1. a) i) Find order and degree of the differential equation $\sqrt{1+\left(\frac{dy}{dx}\right)^2} = 4x^2$

   ii) Solve $2.\dfrac{dy}{dx} = \dfrac{y(x+1)}{x}$ (2 + 2)

   b) Solve $(x-2y)\,dx + x.dy = 0$ (5)

   c) Solve $\cos(x)\,y' + \sin(x)\,y = 2\cos^3 x \, \sin x - 1$, given $y\left(\frac{\pi}{4}\right) = 3\sqrt{2}, \ 0 \le x \le \frac{\pi}{2}$ (6)

2. a) i) Solve $p^2 + 3p - 4 = 0$

   ii) Find general and singular solution of $y = px + p - p^2$ (2 + 2)

   b) Solve $y = 3px + 6p^2 y^2$ (5)

   c) Find orthogonal trajectories of the family $y^2 = 4a(x+a)$, where a is a parameter.

                                                                          (6)

PART – B

3. a) i) Find complementary function of $\left(D^3 - 3D^2 + 9D - 27\right)y = \cos 3x$ where

   $D = \dfrac{d}{dx}$.

   ii) Solve $\left(D^3 - 13D + 12\right)y = 0$ where $D = \frac{d}{dx}$.   (2 + 2)

   b) Solve $\left(D^2 + 4\right)y = \sin^2 x$ where $D = \frac{d}{dx}$ (5)

   c) Solve $\left(D^2 + 9\right)y = x.\cos x$ where $D = \frac{d}{dx}$ (6)

4. a) i) Evaluate $\lim\limits_{x\to 0} \dfrac{x.e^x - \log(1+x)}{x^2}$

       ii) Evaluate $\lim\limits_{x\to 0} \tan x . \log x$ $(2 + 2)$

    b) Verify Cauchy mean value theorem for the function $\sqrt{x+9}$ and $\sqrt{x}$ in $(0, 16)$.

                                                                   (5)

    c) Obtain the Maclaurins expansion of $\sin^{-1} x$ up to terms constraint x^5. (6)

PART – C

5. a) i) If every element of a group G has its own inverse, then prove that G is abelian.

       ii) If a is a generator of a cyclic group G, so is a^{-1}. Prove if it is true. $(2 + 2)$

    b) If a is a generator of a cyclic group G, then prove that $0(a) = 0(G)$ (5)

    c) State and prove Fermat's theorem. (6)

6. a) i) Prove that $a.0 = 0$

       ii) For any two real numbers a and b, show that $|a+b| \leq |a| + |b|$ $(2 + 2)$

    b) If $(1, 2, 3, 4)\, (5, 6, 7)\, (2, 6, 1)\, (4, 7)$ are the cycles. Find disjoint cycle and order of the disjoint cycle. (5)

    c) Find envelope of family of curves $y = mx + \sqrt{a^2 m^2 + b^2}$ where m is a parameter. (6)

PART – D

7. a) i) Find the limit of the sequence $\sqrt{n}\left(\sqrt{n+4} - \sqrt{n}\right)$

       ii) Verify whether n^{th} term of the sequence $\dfrac{\log n}{n}$ is convergent or not $(2 + 2)$

    b) Prove that monotonic decreasing sequence bounded below is convergent. (5)

    c) Prove that limit of a convergent sequence is unique. (6)

8. a) i) Discuss the convergence of the series $1 + \dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27} + \ldots\ldots$

       ii) Show that $\sum\limits_{n=1}^{\infty} \dfrac{1}{2^{n-1}} = 1 + \dfrac{1}{2} + \dfrac{1}{2^2} + \ldots\ldots\ldots + \dfrac{1}{2^{n-1}}$ is convergent $(2 + 2)$

    b) State and prove D'Alemberts Ratio test (5)

    c) Find the sum to infinity $\dfrac{2(3)}{3!} + \dfrac{3(5)}{4!} + \dfrac{4(7)}{5!} + \ldots\ldots\ldots\infty$ (6)

* * * * * * *